Name_____ Date_____

6.2 The Natural Base e
For use with Exploration 6.2

Essential Question   What is the natural base *e*?

So far in your study of mathematics, you have worked with special numbers such as π and i. Another special number is called the *natural base* and is denoted by *e*. The natural base *e* is irrational, so you cannot find its exact value.

1 EXPLORATION: Approximating the Natural Base *e*

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. One way to approximate the natural base *e* is to approximate the sum

$$1 + \frac{1}{1} + \frac{1}{1 \cdot 2} + \frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{1 \cdot 2 \cdot 3 \cdot 4} + \cdots.$$

Use a spreadsheet or a graphing calculator to approximate this sum. Explain the steps you used. How many decimal places did you use in your approximation?

2 EXPLORATION: Approximating the Natural Base *e*

Work with a partner. Another way to approximate the natural base *e* is to consider the expression

$$\left(1 + \frac{1}{x}\right)^x.$$

As *x* increases, the value of this expression approaches the value of *e*. Complete the table. Then use the results in the table to approximate *e*. Compare this approximation to the one you obtained in Exploration 1.

x	10^1	10^2	10^3	10^4	10^5	10^6
$\left(1 + \dfrac{1}{x}\right)^x$						

**6.2** **The Natural Base *e* (continued)**

---

3 **EXPLORATION:** Graphing a Natural Base Function

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use your approximate value of *e* in Exploration 1 or 2 to complete the table. Then sketch the graph of the *natural base exponential function* $y = e^x$. You can use a graphing calculator and the $\boxed{e^x}$ key to check your graph. What are the domain and range of $y = e^x$? Justify your answers.
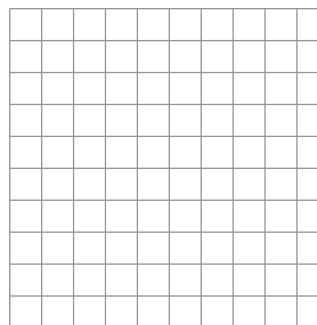
x	−2	−1	0	1	2
$y = e^x$					
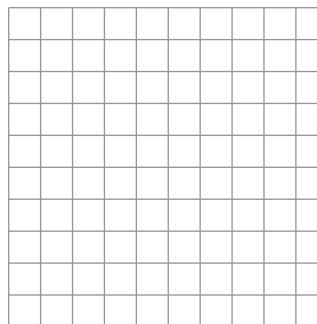
Communicate Your Answer

4. What is the natural base *e*?

5. Repeat Exploration 3 for the natural base exponential function $y = e^{-x}$. Then compare the graph of $y = e^x$ to the graph of $y = e^{-x}$.

x	−2	−1	0	1	2
$y = e^{-x}$					

6. The natural base *e* is used in a wide variety of real-life applications. Use the Internet or some other reference to research some of the real-life applications of *e*.

Name_____ Date _____

6.2 Notetaking with Vocabulary
For use after Lesson 6.2

In your own words, write the meaning of each vocabulary term.

natural base e

Core Concepts

The Natural Base e

The natural base e is irrational. It is defined as follows:

As x approaches $+\infty$, $\left(1 + \frac{1}{x}\right)^x$ approaches $e \approx 2.71828182846$.

Notes:

Natural Base Functions

A function of the form $y = ae^{rx}$ is called a *natural base exponential function*.

• When $a > 0$ and $r > 0$, the function is an exponential growth function.

• When $a > 0$ and $r < 0$, the function is an exponential decay function.

The graphs of the basic functions $y = e^x$ and $y = e^{-x}$ are shown.
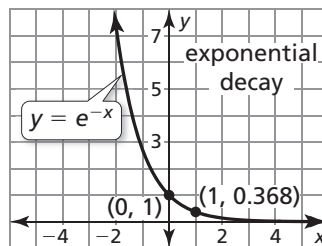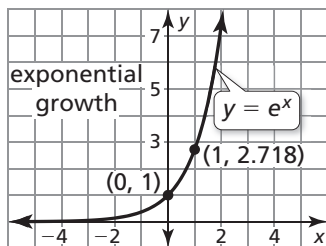
Notes:

6.2 Notetaking with Vocabulary (continued)

Continuously Compounded Interest

When interest is compounded *continuously*, the amount A in an account after t years is given by the formula

$$A = Pe^{rt}$$

where P is the principal and r is the annual interest rate expressed as a decimal.

Notes:

Extra Practice

In Exercises 1–4, simplify the expression.

1. $e^{-9} \cdot e^{12}$

2. $\dfrac{25e^2}{35e^7}$

3. $\left(2e^{-3x}\right)^5 \cdot 2e^{x+1}$
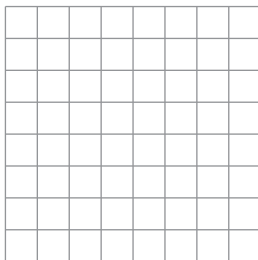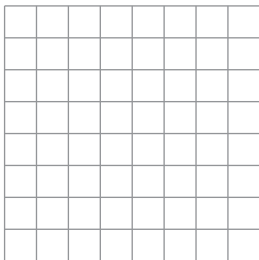
4. $\sqrt[4]{16e^{24x}}$

In Exercises 5–8, tell whether the function represents *exponential growth* or *exponential decay*. Then graph the function.

5. $y = 2e^{-x}$
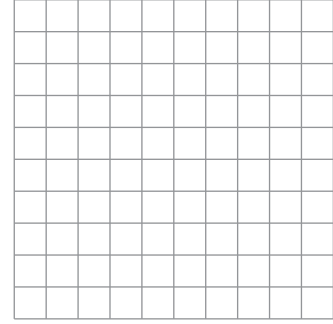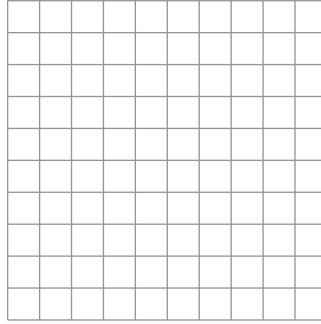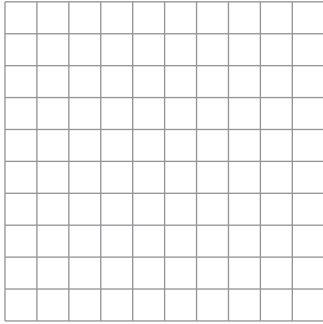
6. $y = 0.75e^{4x}$

7. $y = 5e^{0.25x}$

8. $y = 0.8e^{-3x}$

Name_____ Date _____

In Exercises 9–11, use a table of values or a graphing calculator to graph the function. Then identify the domain and range.

9. $y = e^x - 4$ 　　　　　　**10.** $y = 2e^{x+3}$ 　　　　　　**11.** $y = -e^x + 5$

12. The population of Evans City is currently 48,500 and is declining at a rate of 2.5% each year. You can model the population of Evans City by the equation $P_t = P_c e^{rt}$, where P_c is the current population, P_t is the population after t years, and r is the decimal rate of decline per year. Predict the population of Evans City after 25 years.

13. Your parents will need $25,000 in 10 years to pay for your brother's college tuition. They can invest in an account with an interest rate of 9.8% that compounds continuously. How much should your parents invest today in order to have your brother's full tuition available in 10 years?